20.11.2020

1) i) • $C \subseteq \mathbb{F}_q^4$

• $(0,0,0,0) \in C \Rightarrow C \neq \emptyset$

• $\forall x,y \in C$ için $x+y \in C$ mi?

$x \in C \Rightarrow x = (m,n,p,m+n+p), \quad m,n,p \in \mathbb{F}_q$

$y \in C \Rightarrow y = (m_1,n_1,p_1,m_1+n_1+p_1), \quad m_1,n_1,p_1 \in \mathbb{F}_q$

$$x+y = (\underbrace{m+m_1}_{\in \mathbb{F}_q}, \underbrace{n+n_1}_{\in \mathbb{F}_q}, \underbrace{p+p_1}_{\in \mathbb{F}_q}, \underbrace{m+m_1+n+n_1+p+p_1}_{\in \mathbb{F}_q})$$

$\Rightarrow x+y \in C$

• $\forall x \in C, \forall \alpha \in \mathbb{F}_q$ için $\alpha x \in C$ mi?

$x \in C \Rightarrow x = (m,n,p,m+n+p), \quad m,n,p \in \mathbb{F}_q$

$$\alpha x = \alpha(m,n,p,m+n+p)$$
$$= (\underbrace{\alpha m}_{\in \mathbb{F}_q}, \underbrace{\alpha n}_{\in \mathbb{F}_q}, \underbrace{\alpha p}_{\in \mathbb{F}_q}, \underbrace{\alpha m+\alpha n+\alpha p}_{\in \mathbb{F}_q})$$

$\Rightarrow \alpha x \in C$

$\therefore$ C bir lineer koddur.

ii) $(m,n,p,m+n+p) = m(1,0,0,1) + n(0,1,0,1) + p(0,0,1,1)$

olduğundan

$$G = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 \end{bmatrix}_{3\times 4}$$

iii) G ve C kodundan $k=3$, $d=2$ olarak bulunur.

iv) H ve G arasındaki bağıntıdan

$$H = \begin{bmatrix} q-1 & q-1 & q-1 & 1 \end{bmatrix}$$